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Algorithm for measurement and analysis of authority and influence of users in social and professional networks

Algoritmo para la medición y análisis de la autoridad e influencia de los usuarios en las redes sociales y profesionales

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ABSTRACT

The measurement and analysis of the authority and influence exercised by a person in an organization or social network, be it formal or informal, has been the subject of numerous researches in several fields of science. At present, this phenomenon has taken on greater connotation due to its irruption in the digital space and the importance of having this knowledge for decision-making in spheres such as politics, education and the dissemination of information. In this research, an algorithm was developed for the measurement and analysis of the authority and influence of users in social and professional networks. The study had a mixed approach, with correlational scope and experimental design. A random sample n = 30 specialists was used, which was carried out between May 2019 and October 2020. It was based on the premise that, in order to carry out an adequate measurement and analysis of authority and influence, the structure of the graph must be considered that represents the social network and the interactions that occur between users. As a result, the Total Authority algorithm is developed, a computer tool for the generation of the graph and a case study, which evaluates its relevance, operation and applicability, which shows satisfactory results in its comparison with the HITS algorithm and a sociogram.

Keywords
Algorithm; authority; influence; instant messaging; social media; social network analysis

RESUMEN

La medición y análisis de la autoridad e influencia que ejerce una persona en una organización o red social, sea esta formal o informal, ha sido objeto de numerosas investigaciones en campos diversos de la ciencia. En la actualidad este fenómeno ha tomado mayor connotación debido a su irrupción en el espacio digital y a la importancia que reviste la tenencia de este conocimiento para la toma de decisiones en esferas como la política, la educación y la difusión de información. En esta investigación se desarrolló un algoritmo para la medición y análisis de la autoridad e influencia de los usuarios en las redes sociales y profesionales. El estudio tuvo un enfoque mixto, con alcance correlacional y diseño experimental. Se utilizó una muestra aleatoria n=30 especialistas, que se realizó entre mayo de 2019 y octubre de 2020. Se partió de la premisa de que, para llevar a cabo una adecuada medición y análisis de la autoridad e influencia, se debe considerar la estructura del grafo que representa la red social y las interacciones que se producen entre los usuarios. Como resultado, se desarrolla el algoritmo Autoridad Total, una herramienta informática para la generación del grafo y un caso de estudio, el cual evalúa su pertinencia, funcionamiento y aplicabilidad, lo que evidencia resultados satisfactorios en su comparación con el algoritmo HITS y un sociograma.

Palabras clave
Algoritmo; análisis de redes sociales; autoridad; influencia; mensajería instantánea; redes sociales

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Introduction

Interaction between people and the formation of active social networks is a fact that dates back to the emergence of the human species and the creation of communities. Since then, people have felt the need to interact with each other, either because of the existence of bonds of familiarity or friendship, interests, ideologies or common habits, or because of the need for human beings to relate to each other.

The emergence and rapid development of the Internet and information and communication technologies (ICTs) revolutionized the way in which, until then, social networks had functioned. The different paradigms that have evolved the web, such as the social web, the semantic web, the ubiquitous web, the internet of things and affective computing, have also reconfigured the way in which social networks behave as virtual communication spaces (Del Prete & Pantoja, 2020). This is why social networks currently stand as one of the most booming areas of research and investigation by various sciences, as diverse and separate in object of study as sociology, psychology, mathematics or computational sciences. Nowadays, individuals communicate continuously through some technological device or medium, and more and more people have access to the Internet for this purpose.

In mathematics, this structure of social networks is represented by means of graph theory, in which the network is made up of nodes, which correspond to actors or individuals, and edges, which are the relationships established between individuals. Internet social networks can connect people and help them maintain their relationships throughout their lives. Currently, some of the most popular media are Facebook, WhatsApp, YouTube or Instagram, which can be used by individuals to build and expose their identity online.

An important component of social media is instant messaging. Its use allows active communication between two or more users, by means of electronic devices connected to the network. This type of messaging is characterized by being conducted in real time and is based on the exchange of text messages (Sun, Lin, Wu, Zhou & Luo, 2018). Instant messaging has become an inevitable part of the habits of the majority of the population, forging a new way of understanding communication and new expectations around the exchange between sender and receiver. This communicative process has evolved significantly, so that any type of information can now be shared, such as text, videos and photos, and so on (Andújar-Vaca & Cruz-Martínez, 2017).

Instant messaging allows the exchange of messages that are written and read at the same time, despite the distance between users (Sewall, Bear, Merranko & Rosen, 2020). With the contact groups of these users, a social network can be formed. The magnitude of its size depends on the number of contacts that each user has.

In many institutions the instant messaging service is used as an informal means of communication among workers. According to various authors, in several cases, influence is determined to a greater extent by communication that is established informally than by that carried out by formal means (Sadri, Ukkusuri and Ahmed, 2021). For this, companies can use social network analysis (SNA), which is a valuable source of information to identify the most influential workers or individuals within an organization or context.

SNA is a potential and currently growing area of research, originating in sociology. It comprises numerous tools, metrics and methods for modeling, measuring and analyzing the relationships and behaviors that are established between people, to support data-driven decision making (Perez, 2016). The analysis of these relationships reflects the ability of some people to influence the behavior of others. This behavior is referred to as social influence (Peng, Yang, Cao, Yu & Xie, 2017). The study of social influence is not only difficult to measure, but the criteria for influence vary depending on the context and measurement objectives. For Peng *et al*. (2017), the analysis of social influence is one of the most important technologies in service and information companies, because its efficient use has a positive impact on organizational culture and decision making, as well as on adequate strategic planning and management to obtain competitive advantages.

Social influence analysis

A user's social influence is found in his or her ability to induce reactions in other users (Saggu and Sinha, 2020). This occurs when one person's behavior, perceptions, emotions, and opinions are affected by another. The primary sign of an influential interaction is when a user's action causes cascading reactions in other users. The level of authority possessed by a user of a social network is largely determined by his or her hierarchy and credibility within it (Perez, 2016; Xiao, Wang & Chan-Olmsted, 2018). The more authority a user possesses, the greater his or her influence.

Likewise, social influence and related phenomena such as leadership and social power are intrinsic to any human organizational process. In work groups, social influence affects how decisions are made, how work is carried out, as well as the organizational climate. Interactions and communication are central to social influence (Flache *et al*., 2017; Mahmoodi, Bahrami & Mehring, 2018; Sadri *et al*., 2021).

Identifying the most influential people (influencers, from the English language, as it is commonly known) has many applications for any organization. Studying patterns of authority, leadership, hierarchy, influence, and information propagation helps to understand why certain trends or innovations are adopted more quickly than others (Das, Kamruzzaman & Karmakar, 2019). Social influence analysis (SIA) is a component of SNA that studies how to model the process of influence diffusion in networks and how to propose an efficient method to identify a group of target nodes in a social network. Knowing this information would make it possible to make decisions related to determining the leadership, strength or cohesion of social groups and knowledge of the key players in the dissemination of information.

In this sense, current researches such as those conducted by Gil-Quintana, Malvasi, Castillo-Abdul and Romero-Rodríguez (2020) and Gil-Quintana, Santoveña-Casal and Riaño (2021). The authors analyze the importance of the internet, technologies and social networks, as well as influencers in the propagation of information, in an increase in social influence, leadership, learning and collaboration. Its rise in recent years has been marked by the increasing irruption of ICTs in these unconventional contexts, based on their impact on increasing competitive performance in all spheres of society (Alvarado, 2021; Pérez, López-Torres & Morejón-Valdés, 2021).

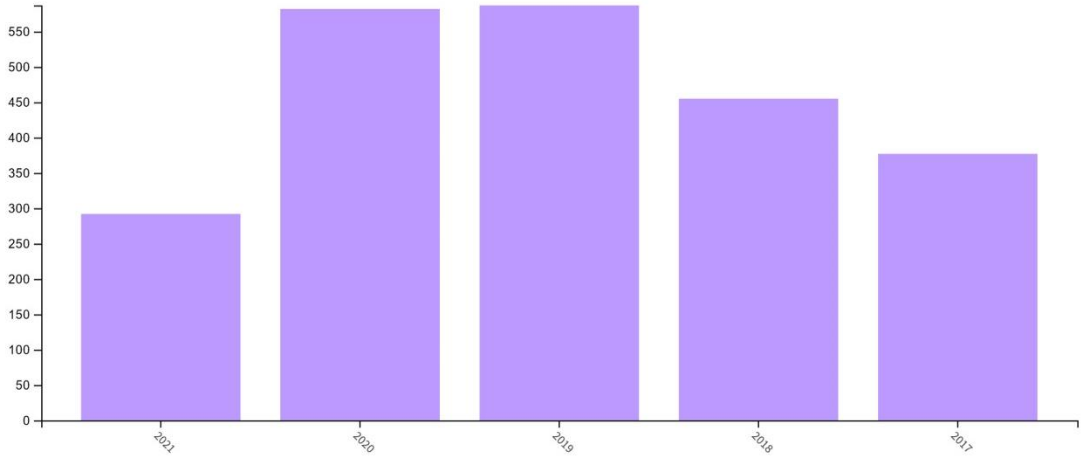
Precedents and substantiation

Social influence analysis is increasingly employed and used in various sectors of society, such as medicine, sports, politics, education and aeronautics. Also, numerous techniques are applied for data and information analysis in professional and social networks from various perspectives and approaches, as evidenced by some of the consulted research (Andújar-Vaca & Cruz-Martínez, 2017; Das *et al*., 2019; Huang, Wang & Chen, 2019; Pérez, 2016; Sewall *et al*., 2020; Zhao, Xu, Song, Lee, Chen & Gao, 2018). For this, internet social networks have been formed as the most important link to access the digital global world, to share information and use it for efficient processing and employment in decision making (Peng *et al*., 2017; Wajahat, Nazir, Akhtar, Qureshi, Razaque & Shakeel, 2020).

In this sense, the main questions in social influence analysis studies include: who influences whom, who is influenced, and who are the most influential users, which are of great use in fields such as viral marketing, online recommendation, healthcare communities, and expert discovery (McCraty, 2017; Pérez, Vázquez, Valdés & Fajardo, 2016; Sun *et al*., 2018). According to research by Colomo (2017), Del Prete & Pantoja (2020) and Giacomucci (2020), social influence analysis can help to understand people's social behaviors, provide theoretical support for decision making, influence public opinion and promote change.

Knowing the most influential actors in social networks has been of interest to the scientific community, as stated by Kim & Hastak (2018) and Sadri *et al*. (2021). This is why studies related to this area of knowledge have had a high growth from 2010 to date. A bibliometric analysis in the Web of Science (WoS) with the terms “social network analysis” and “influence analysis and social media” showed their use and evolution. Figure 1 shows these data, which are grouped by year of publication. In the documentary analysis, for the key terms analyzed, a total of 2,293 results were found in the period from 2017 to 2021.

Figure 1. Sustained growth of publications in WoS on "social network analysis" and "influence analysis and social media"



Source: developed by the author with data from Clarivate Analytics.

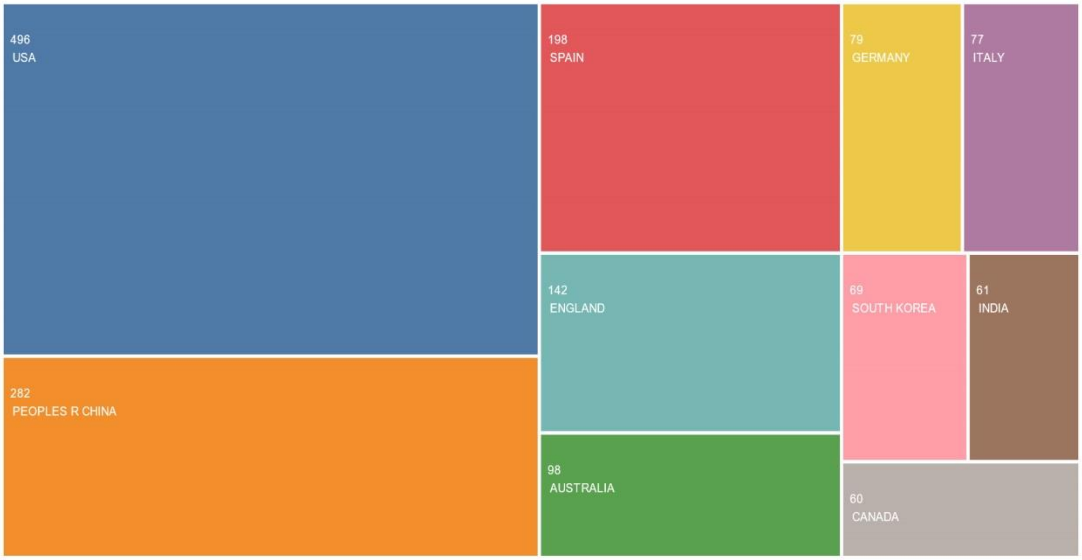
Likewise, in the documentary analysis carried out, using the databases Web of Science Core Collection, SciELO Citation Index, Derwent Innovations Index, Russian Science Citation Index and KCI-Korean Journal Database, it is evident that the main areas of research using "social network analysis" and "influence analysis and social media" are computer science, economics and business, mathematics, psychology, communication, behavioral sciences, sociology and psychology, among other areas (see figure 2). The above shows the relevance and the need to delve deeper into this topic due to the importance of the analysis of social influence in organizations at a global level.

Similarly, of the total of 2293 research results, 21.63% are created by the United States, which is the country that presents the most results in the analysis of social networks and their influence. Globally, China is the second country with 12.29% (see figure 3). In addition, of all the scientific production created in the 2017-2021 period, 94.65% is for research articles, 4% for review articles and 1.5% for patented innovation results.

Figure 2. Analysis of the use of "social network analysis" and "influence analysis and social media" according to the area of research and application

Source: developed by the author with data from Clarivate Analytics.

Figure 3. Analysis of the use of "social network analysis" and "influence analysis and social media" according to geographic region



Source: developed by the author with data from Clarivate Analytics.

Methodology

The objective of this research was to develop an algorithm for the measurement and analysis of the authority and influence of users in social and professional networks. The hypothesis of the research was: the algorithm developed is valid to adequately measure and analyze the authority and influence exercised by users in social and professional networks. For this, a case study was carried out to evaluate the performance and validity of the algorithm developed. It was carried out in a university organizational context governed by formal and informal relationships, where the results obtained with the HITS algorithm and a sociogram were compared. Satisfactory results were expected in the evaluation of relevance and applicability.

The research was conducted between May 2019 and October 2020. Its approach was mixed, qualitative and quantitative. For this, scientific methods such as observation, modeling and documentary analysis were used, which included a bibliometric analysis of the current state of the subject and a retrospective analysis between 2010 and 2021. The sources consulted have a high percentage of topicality, 90.48% (2017-2021). They were primary sources, obtained from journals in high-impact databases.

The research has a correlational scope, while the design is experimental and cross-sectional. The sample used is random n=30, with which a case study was conducted to evaluate the proposed solution developed, taking into account that the purpose of the study was to develop an algorithm for the measurement and analysis of the authority and influence of users in social and professional networks.

Domain metrics and concepts

- **Degree centrality:** it is counted as the number of neighbors or connected nodes that a certain node has in a network scheme (Huang *et al*., 2019; Kim & Hastak, 2018). Degree centrality is the number of edges incident on a given vertex. Weighted degree centrality is defined similarly, but the weights of the incident edges are summed.

- **Betweenness centrality:** is defined as the level of influence a node or actor has within a network, marked by the minimum number of route in which it is seen to be present (Kim & Hastak, 2018). It is a very important metric because it can be used to identify information brokers in the network or nodes that can connect separate groups. This is why a node with high broker centrality exerts high influence as part of a network (Huang *et al*., 2019; Tuğal & Karcı, 2019).

- **Closeness centrality**: consists of the average of the distances from a vertex to all vertices, by considering that small values manifest higher importance (Guan, Li, Xing, Li & Liang, 2020). It represents the nodes that, even having few links or connections, favor reaching any other node in the network faster.

In the research, the closeness centrality metric is implemented, where the closeness value is normalized by considering the number of users in the network (see formula 1). SN is the normalized closeness value, N the number of nodes and S the total raw closeness value. Thus, SN is not an inverse metric, so that nodes with higher values are more central.

$$SN = (N - 1)/S \tag{1}$$

- **Eigenvector centrality:** this is defined as recursive centrality. A node can be categorized as a central node if it has neighboring nodes or connected nodes that have, in turn, good centrality as part of the network (Kim & Hastak, 2018).

- **PageRank:** this is Google's search algorithm. It works by counting the number and quality of hits to a page to determine its importance. It constitutes a variation of eigenvector centrality. In this algorithm, each vertex of the network acquires a value from its neighbors. Unlike eigenvector centrality, a vertex does not acquire the total importance from its neighbors. Instead, this importance is divided equally among its direct connections. PageRank was introduced to account for the positioning of websites on the Internet (Tortosa, Vicent & Yeghikyan, 2021).

- **HITS:** allows detecting hubs and authorities within the networks of links formed between web pages. Hubs are defined as websites that have outbound links to other pages, and authorities are those widely referenced by other websites (Kanathey, Thakur and Jaloree, 2018). The notion of hub expresses the quality of a website and the notion of authority establishes the quality of a website as a resource itself. This importance or relevance is determined according to the query made by the user. Thus, each web page is assigned hub and authority scores to provide the user with the information he required, which has been requested in the form of a query and processed by search engines (Kanathey *et al*., 2018).

- **Sociogram:** the concept comes from sociology, as a science that studies societies and human behavior. It is defined as a technique that allows the formal representation of existing social relationships between people in a given context (McCraty, 2017). In graph theory, it enables the representation of relationships, based on nodes that constitute people and edges, which respond to the relationships that are evident between them (Hurtado, Leiva & Villalobos, 2018).

Mathematical formulas

- **Normalization by the largest element:** this is a mathematical technique of social network analysis, which divides all the elements per column by the largest element of each of the columns. This variant ensures that the result is in the interval [0,1] and that proportionality is maintained. The formula is shown below (Bellver and Martinez, 2012) (see formula 2).

$$\boldsymbol{X_i normalized} = \frac{\boldsymbol{X_i}}{\mathbf{max}\,\boldsymbol{X_i}} \tag{2}$$

- **Weighted Average (WA).** Its definition is as follows: To an aggregation operator of type WA is associated a vector of weights α, where $\alpha \in [0,1]$, while $\sum_1^n \alpha_i = 1$. In formula 3 below, α_i is the importance attached to the data source X_i:

$$\boldsymbol{WA(X_1, \dots, X_2) = \sum_{i=1}^{n} X_i \alpha_i} \tag{3}$$

Correlation coefficients

- **Correlation:** observed when two variables evidence the existence of a relationship, either positive linear or negative linear, which does not imply the existence of causality. A common way of representing the correlation between two variables is by means of a scatter diagram.

- **Pearson's coefficient** $(\boldsymbol{r})$**:** this is commonly known as the linear correlation coefficient. It calculates the degree of relationship that can be carried out between two quantitative variables, as part of a sample. The formula for Pearson's correlation coefficient is calculated as follows (Triola, 2009, p. 522) (see formula 4).

$$\boldsymbol{r} = \frac{\boldsymbol{n \sum xy - (\sum x)(\sum y)}}{\sqrt{\boldsymbol{n(\sum x^2) - (\sum x)^2}} - \sqrt{\boldsymbol{n(\sum y^2) - (\sum y)^2}}} \tag{4}$$

Where n represents the number of pairs of data present.

- **Spearman's coefficient** $(\boldsymbol{r_s})$**:** also called rank correlation coefficient, it is a non-parametric contrast that works with data from a sample, with the purpose of demonstrating whether there is an association or correlation relationship between two variables. Because it is calculated from ranges of data and not from the values themselves, it is less sensitive to outliers than Pearson's coefficient (Triola, 2009). For its calculation, formula 5 is taken into account if there are no ties, and formula 6 if there are ties.

$$\boldsymbol{r_s = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}} \tag{5}$$

Where n represents the number of ranks and d the difference between pairs of ranks, whereby the highest and lowest ranks are subtracted.

$$r_s = \frac{n\sum xy - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2}\sqrt{n(\sum y^2) - (\sum y)^2}} \tag{6}$$

Both coefficients return a value ranging from -1 to 1. Values close to both extremes (negative or positive) affirm the existence of a strong correlation between variables, linear negative or linear positive, as the case may be. On the other hand, values close to 0 indicate no correlation or almost no correlation (Triola, 2009).

Results

The section develops the total authority (TA) algorithm, which allows measuring the influence and authority of a user in social and professional networks in a value between zero and one hundred [0,100]. It is determined by the hierarchy and credibility that users have. This is why the algorithm developed is comprised of two other algorithms, for further measurement and analysis:

- **Structural authority (SA):** measures the hierarchy of IM network users by representing it as a graph and calculating the value of the graph centrality metrics.

- **Reaction Authority (RA):** measures credibility by considering the immediacy with which a written message receives responses in IM and the number of responses.

A user's TA is calculated by adding its SE and its AR (see formula 7).

$$TA = SA + RA \tag{7}$$

Structural Authority (SA)

The SA of a node is calculated from its centrality within the network. The degree centrality in a network, despite being an old and primordial term in the analysis of social networks, is currently used in various fields, as assured in their research by several authors (Riquelme, González-Cantergiani, Molinero & Serna, 2018; Zhao *et al*., 2018).

For the calculation of the SA, the degree centrality (D) is considered, because it represents the popularity (amount of contacts) of the user, and the closeness centrality (C), because it allows identifying how quickly this user can relate to others. Likewise, the intermediation centrality (B) is considered, because it allows detecting those users who connect the different communities formed in the network, and the PageRank (PR), because it makes it possible to identify the importance of the user based on the relevance of his contacts. The values of these metrics are unified by using the normalization technique by the largest element and then combined using the weighted average aggregation operator. After applying the aggregation operator, the result is multiplied by 100 so that the SA values are in the interval [0,100]. The general formula for calculating the SA is as follows (see formula 8).

$$\boldsymbol{SA = (\sum_{i=1}^{n} X_i normalized * \alpha_i) * 100} \qquad (8)$$

X constitutes the vector that stores the centrality metrics, $X = \langle D, C, B, PR \rangle$ and α is the vector of the weights that is applied to each metric to determine how much it contributes to the SA. $\alpha = \langle \alpha_D, \alpha_C, \alpha_B, \alpha_{PR} \rangle$, is determined at the discretion of the expert who will use the formula and may vary when considering the data environment where it will be applied. Clearing the values of X results the extended formula (see formula 9).

$$\boldsymbol{SA = \left(\frac{D}{\max D} * \alpha_D + \frac{C}{\max C} * \alpha_C + \frac{B}{\max B} * \alpha_B + \frac{PR}{\max PR} * \alpha_{PR}\right) * 100} \qquad (9)$$

For the calculation of the SA in a $G(V, E)$ graph the following algorithm is defined (table 1).

Table 1. Algorithm to calculate the AE

Inputs	$V(G)$ where each node v_i has as attributes the values of the metrics of the vector X, array of weights α
Outputs	$V(G)$ with its nodes v_i enriched with the value of its SA as attribute
Beginning Start $AE := 0$ For each $v_i \in V(G)$ $SA := \left(\frac{D}{\max D} * \alpha_D + \frac{C}{\max C} * \alpha_C + \frac{B}{\max B} * \alpha_B + \frac{PR}{\max PR} * \alpha_{PR}\right) * 100$ $v_i.SA := SA$ End for Resume $V(G)$ End	

Reaction Authority (RA)

This algorithm is based on the criterion that when someone receives a message from another person that he/she considers important, he/she responds to it in the shortest possible time. In other words, for a user to have authority over another user in instant messaging, they must receive a response to their messages within an immediate or short time interval, which is denoted by the letter I. The interval may vary depending on how they use instant messaging.

The interval may vary depending on the use of instant messaging by the people analyzed, so it should be considered for its definition.

For two users u and v, the RA of a message sent by u to v (RAM_{uv}), at a time instant t_0, is calculated by counting the number of replies to u's message in a time interval including from t_0 to $t = t_0 + I$. Its formula (10) and the pseudocode of the developed algorithm are shown below (see table 2).

$$\boldsymbol{RAM_{uv}} = \sum \boldsymbol{R_{vu}} \tag{10}$$

Where R_{vu} is a received response, consisting of a message where v is the sender, u is the receiver and the date of the message ($date_R$) is in the time interval ($t_0 < date_R < t_0 + I$).

Table 2. Pseudocode of the algorithm to calculate the RA of each node of a graph G

Inputs	A directed graph $\mathrm{G(V,E)}$ where each element of E represents the relation "WRITES_TO" $((v_{source}) - [WRITES\_TO] -> (v_{target}))$, an integer ($I$) representing the time range in which reply messages will be counted
Output	Graph $G(V,E)$ with its nodes $V(G)$ enriched with the value of its RA as attribute

Beginning

1) $RA_u := 0$
2) $neighborList := null$
3) For each $u \in V(G)$
   3.1. neighborList $:=$ $\mathrm{u.getNeighbors}$
   3.2. for each $v \in neighborList$
      3.2.1. $SumRAL_{uv} += CalculateRAL_{uv}(u, v, I)$
   3.3. End for
   3.4. $RA_u := {SumRAL_{uv}}/{ListNeighbors.quantity}$
   3.5. $u.RA := RA_u$
4) End for
5) $Normalize\ RA$

End

The RA of a user u over another user v consists of the average of the RAM_{uv} of each message sent by u to v. This is known as the local reaction authority (LRA) of u over v (LRA_{uv}). Its formula (11) and the pseudocode of the developed algorithm are shown below (see table 3).

$$\boldsymbol{LRA_{uv}} = \frac{\sum \boldsymbol{RAM_{uv}}}{\boldsymbol{m}} \tag{11}$$

Where m is the number of messages u has sent to v.

Table 3. Pseudocode of the algorithm to calculate the LRA_uv

Inputs	Two neighboring nodes u, v and an integer (I) that represents the time range in which the response messages will be counted
Output	LRA_{uv}

Beginning
1) $LRA_{uv} := 0$
2) $sentList := u.sentMessages(v)$
3) For each $m \in sentList$
   3.1.1. $SumRAM_{uv} += CalculateRAM_{uv}(m, I)$
   3.2. End for
4) $LRA_{uv} := {}^{SumLRA_{uv}}/_{sentList.quantity}$
5) Return LRA_{uv}

Return G

End

The total RA of a user u (RA_u), is calculated by adding all its LRA. Its formula (12) and the pseudocode of the developed algorithm are shown below (see table 4).

$$\boldsymbol{RA_u = \sum_{i=1}^{n} LRA_{ui}} \qquad (12)$$

Where n is the number of neighbors of u; i is the neighbor of u to be analyzed in each case.

Table 4. Pseudocode of the algorithm to calculate the RAM_u

Inputs	Edge m that represents a message sent by u to v and an integer (I) that represents the time range in which the response messages will be counted
Output	RAM_{uv}
Description	Count messages from v to u in the time range specified by I

Beginning
1) $RAM_{uv} := 0$
2) $msg.List := m.receiver_v.sentMessages(u)$
3) For each $msg \in msg.List$
   3.1. If $m.date < msg.date < m.date + I$
      3.1.1. $answers++$

3.2. End for 4) Return *answers* End

The last step to calculate the AR is to normalize its values by using the Normalization technique by the largest element and then multiply them by 100 so that they are in the interval [0,100] and can be used to calculate the Total Authority (TA). Its formula (13) and the pseudocode of the developed algorithm are shown below (see table 5).

$$\boldsymbol{RA_{normalized}} = \frac{\boldsymbol{RA_u}}{\mathbf{max}\,\boldsymbol{RA}} * \mathbf{100} \qquad (13)$$

Table 5. Pseudocode of the algorithm to normalize the RA of each node of a graph G

Inputs	A directed $\mathrm{G(V,E)}$ where each node $V(G)$ has its RA computed
Output	Graph $G(V,E)$ with the normalized RA of each node $V(G)$
Beginning 1) $\max RA := \mathit{Find\ the\ largest\ RA}$ 2) For each $u \in V(G)$ 2.1. $RA_{normalized} := \left({}^{u.RA}/_{\max RA}\right) * 100$ 2.2. $u.RA := RA_{normalized}$ 3) End for End	

Case study to evaluate the performance of the developed algorithm

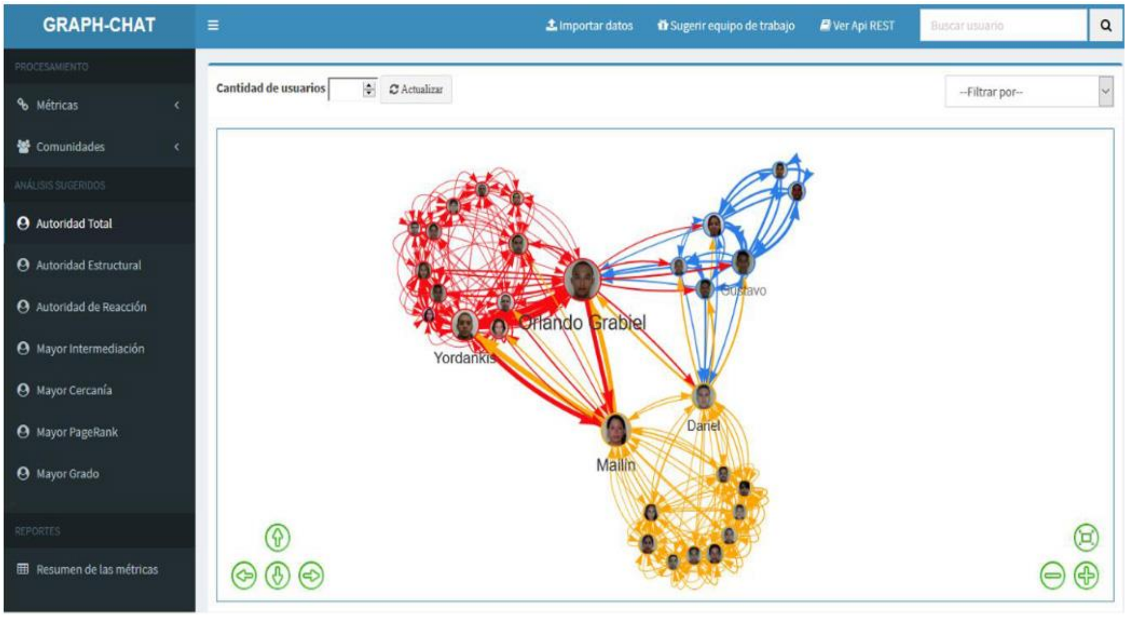
To verify the results, a case study was carried out on 30 teachers from Faculty 4 of the University of Informatics Sciences (UIC), located in the province of Havana, Cuba. The study consisted of representing in a graph the relationships established among these people in the instant messaging of the UIC. Then, the influence value of each one was calculated, considering the authority algorithm developed in the research (total authority), the authority value provided by the HITS algorithm and the influence value determined from the application of a sociogram.

Finally, the correlation between these values was determined using Pearson's and Spearman's correlation coefficients, and precise relationships between them were identified. In this way it is possible to evaluate the fulfillment of the research hypothesis, by being able to compare the results obtained in each case, between the developed algorithm, the HITS algorithm and a sociogram, the latter two being established techniques. It is desired to demonstrate that the algorithm developed is valid to adequately measure and analyze the authority and influence exercised by users in social and professional networks

Obtaining the data and calculating the influence value

To represent the graph and to calculate the TA, GRAPH-CHAT, a tool developed by the authors was used to obtain the graph which represents the social network (Del Castillo, 2018) (see figure 4).

Figure 4. Main interface of the GRAP-CHAT client



The input data for the tool were obtained from the conversation histories of those involved, with their prior authorization. The HITS authority value was calculated with the help of the Gephi tool (Wajahat *et al*., 2020), which was configured to connect to the GRAPH-CHAT database and, in this way, the graph could be accessed. The sociogram applied to these individuals was designed by a sociology expert. Table 6 shows the values calculated for each of the metrics applied and the algorithms developed in the study.

Table 6. Calculated values of TA, SA, RA, Authority through HITS and influence through the sociogram for each user

User	Structural authority	Reaction authority	Total authority	HITS	Sociogram
1	30,83	0,00	15,42	34,29	22,34
2	30,45	0,00	15,22	31,26	20,21
3	36,90	0,00	18,45	36,45	26,60
4	42,76	47,63	45,19	21,58	52,13
5	63,09	21,70	42,39	75,13	48,94
6	37,36	0,00	18,68	76,37	26,60
7	42,76	67,09	54,92	21,58	62,77
8	30,48	43,15	36,82	20,75	41,49
9	37,36	0,00	18,68	76,37	26,60
10	30,48	11,46	20,97	20,75	24,47
11	30,98	0,00	15,49	33,66	22,34
12	37,36	20,68	29,02	76,37	37,23
13	37,36	28,64	33,00	76,37	41,49
14	22,69	26,91	24,80	4,61	28,72
15	28,62	0,00	14,31	30,53	19,15
16	37,15	5,73	21,44	77,05	29,79
17	70,82	47,30	59,06	100,00	63,83
18	22,69	29,97	26,33	4,61	29,79
19	35,54	15,08	25,31	37,40	32,98
20	37,01	0,00	18,51	77,10	26,60
21	100,00	100,00	100,00	69,92	100,00
22	32,85	23,99	28,42	31,79	35,11
23	50,98	35,75	43,37	56,74	53,19
24	38,80	24,67	31,74	39,31	36,17
25	42,77	95,02	68,89	47,71	77,66
26	45,73	22,44	34,08	50,78	41,49

User	Structural authority	Reaction authority	Total authority	HITS	Sociogram
27	34,80	71,60	53,20	70,24	62,77
28	29,10	0,00	14,55	28,88	20,21
29	37,36	0,00	18,68	76,37	26,60
30	37,36	29,66	33,51	76,37	42,55

Comparing the calculated values and analysis of the results

To compare the calculated values, a correlation analysis was carried out using the IBM SPSS Statistics 22 statistical package, and two matrices were obtained. The first matrix is shown in table 7, where the values represented are the Pearson Coefficient between each pair of variables. Likewise, table 8 shows the data obtained after the application of Spearman's correlation coefficient.

Table 7. Representation of Pearson's correlation values

	Structural authority	Reaction authority	Total authority	HITS	Sociogram
Structural authority		0,5551	0,7966	0,5097	0,7797
Reaction authority	0,5551		0,9450	0,0380	0,9453
Total authority	0,7966	0,9450		0,2280	0,9936
HITS	0,5097	0,0380	0,2280		0,2767
Sociogram	0,7797	0,9453	0,9936	0,2767	

Source: developed by the author with data from IBM SPSS Statistics 22.

Table 8. Representation of Spearman's correlation values

	Structural authority	Reaction authority	Total authority	HITS	Sociogram
Structural authority		0,4399	0,6931	0,5751	0,7284
Reaction authority	0,4399		0,9347	-0,0966	0,9084
Total authority	0,6931	0,9347		0,1477	0,9866
HITS	0,5751	-0,0966	0,1477		0,2423
Sociogram	0,7284	0,9084	0,9866	0,2423	

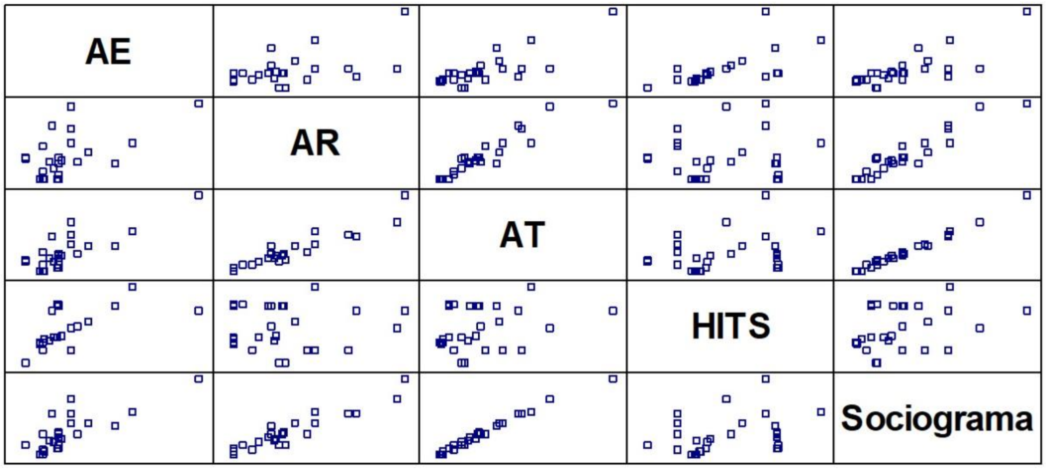
Source: developed by the author with data from IBM SPSS Statistics 22.

When the values obtained from the correlation coefficients were analyzed, it was observed that there is a relatively weak relationship between TA and the authority score given by HITS ($r = 0.2280$ and $r_s = 0.1477$). However, the relationship between TA and HITS authority is moderately strong ($r = 0.5097$ and $r_s = 0.5751$). This is because both metrics consider only the representative network structure to be calculated. The HITS assigns its authority value based on the characteristics of the existing relationships between nodes, while the AE consists of the combination of the main centrality metrics (Degree, Closeness, Intermediation and PageRank) so it also takes into account the quality and quantity of the links, in addition to the strategic position of the node. On the other hand, HITS does not analyze the reactions provoked by IM users and that is why its correlation with RA is almost null ($r = 0.0380$ and $r_s = -0.0966$).

Another aspect that stood out in the analysis was the significant correlation between the influence value obtained with the sociogram and the TA ($r = 0.9936$ and $r_s = 0.9866$) as well as the weak correlation between the sociogram and the HITS authority ($r = 0.2767$ and $r_s = 0.2423$). The analysis allows us to determine that, in order to carry out an adequate measurement and analysis of authority and influence, the structure of the graph representing the social network and the interactions that occur between users must be considered. Furthermore, it demonstrates the relevance, effectiveness and usefulness of TA to identify influential users in instant messaging and its superiority in this context over the HITS algorithm.

Finally, figure 5 shows the analyses performed by means of scatter plots. This clearly shows the linear relationship between TA and the sociogram, since the points that plot the correlation between these two variables are grouped together and form a straight line. Likewise, the points that plot the correlation between TA and HITS and between HITS and the sociogram appear dispersed, which reaffirms the existence of a low correlation and almost null relationship between the set of data analyzed. The above indicates the relevance and validity of the algorithm developed to measure the authority and influence of users in this environment.

Figure 5. Dispersion graphs obtained for each pair of variables



Source: developed by the author with data from IBM SPSS Statistics 22.

Discussion

In this research, an algorithm was developed for the measurement and analysis of the authority and influence of users in social and professional networks, called total authority (TA), which is composed of two algorithms that allow to deepen and characterize social influence in an optimal way: the structural authority algorithm (SA) and the reaction authority algorithm (RA). These were presented on the basis of pseudocodes, so that the research can be applicable and reproducible in other contexts where the environmental conditions allow it.

Subsequently, a case study was applied to evaluate the performance and validity of the developed algorithm. For this, the results obtained were compared with the HITS algorithm and a sociogram (Hurtado *et al*., 2018; Kanathey *et al*., 2018). Both tools present a vast documentation, are well known and widely used nowadays. To carry out the comparison of the results, statistical techniques such as Pearson's correlation and Spearman's correlation were used (Triola, 2009). The IBM SPSS Statistics package, version 22, was used to analyze the data obtained. The use of both correlation coefficients showed statistically significant results in favor of the algorithm developed.

The results obtained confirm the fulfillment of the research hypothesis, by demonstrating that the developed TA algorithm is valid to adequately measure and analyze the authority and influence exercised by users in social and professional networks. It can be used to support organizational decision making, in relation to the measurement of social influence, the analysis of the interaction of people in formal and informal networks, as well as in the calculation of the authority exercised by certain individuals.

Analysis of these aspects is considered of great importance in a current context, which is governed by the knowledge society (KS), internet and the fourth industrial revolution, where information and its use for decision making acquires a very high value (Jennex, 2017; Philbeck and Davis, 2018; Schwab, 2017); however, Jayles *et al*. (2020) warn that in this KS in which large amounts of data are continuously generated, not all the information that is disseminated is correct, which can affect collective wisdom and organizational climate. Due to the above, it is important for organizations to appropriate tools for an adequate analysis and processing of the levels of social influence.

Likewise, as Ryu & Han (2021) refer, the analysis of social influence not only has a positive connotation on the existing information in the environment and its use to make appropriate decisions that impact organizations, but has increased the political influence and economic value of social network systems. This is due to society's need to have control over opinion leaders, as well as to influence people's decisions, know their preferences and foresee events, based on the social influence exerted by these leaders (Zhang & Gong, 2021).

Several authors give merit to research in this field of application and recognize its importance in the continuous study of human relations and in the improvement of organizational behavior. This is the case of Mahmoodi *et al*. (2018), who state that people seek to improve their decisions through social interaction, in which they obtain advice and other points of view that are subsequently applied in their daily lives. The case study analyzed corresponds to a collective of UCI research teachers, where their interaction takes place around their formal relationships and the need to collaborate in order to have a superior professional performance (Del Castillo, 2018).

On the other hand, Peng *et al*. (2017) warn of the need to analyze the social influence exerted by one influential user on another in mobile social networks, both directly and indirectly. This is due to the accelerated evolution of technologies and digital web platforms, which impose new trends and prediction models. However, new complexities are detected in their analysis, such as the need to develop social influence models for platforms with these characteristics (Kanathey *et al*., 2018).

To Flache *et al*. (2017) social influence reduces the differences that exist between people. Due to the above, the continuous creation of models that allow understanding why and under what conditions an influence can coexist between people becomes necessary, even in conditions where the diversity of attitudes, beliefs and behaviors is very different. In this sense, the consulted authors explore the possible effects of social networks in the continuous polarization suffered by society.

In the context of the research, influencers constitute a key element in the propagation of information, in the generation of authority levels and in social influence (De Veirman, Cauberghe & Hudders, 2017). Their principles of behavior as a social entity were considered in the development of the TA algorithm. Dissimilar research has been developed where these actors are recognized as information brokers and opinion leaders (Delbaere, Michael & Phillips, 2021). In addition, some researchers suggest that the uniqueness and originality of these individuals constitute determining elements for a user to be recognized as an opinion leader (Casaló, Flavián & Ibáñez-Sánchez, 2018; Oliveira, García & Vivacqua, 2021).

On the other hand, Ryu & Han (2021) state that in recent years, influencers have attracted attention, not only because they influence people's behavior, but also in the decisions they make and in their opinion formation. For this reason, the authors consider that their reputation is what makes them worthy of this influence, determined by four distinctive dimensions: authenticity, influence, communication skills and experience. Casaló *et al*. (2018) concurred with some of these dimensions above. In addition, Xiao *et al*. (2018) incorporate credibility as another distinctive factor of these individuals and agree with the need for the existence of high social influence.

All the findings presented and aspects discussed in this section were part of the theoretical bases, scientific foundations and practical elements considered in the development of the research and algorithm for further applicability and relevance. The discussion and comparison of positions of relevant authors and updated research in the area of knowledge favored an optimal contextualization of the analyzed phenomenon.

Conclusions

After conducting the research, it is concluded that in order to carry out an adequate measurement and analysis of the authority and influence of users in social and professional networks, by means of graph representation, the analysis of TA, the result of the sum of SA and RA, should be considered. In addition, the levels of influence possessed by the workers of the organizations in an informal network of social interaction, such as instant messaging, should be known, which makes it possible to identify the most popular users, the most followed, the possible leaders of different existing social groups, those who are the bridge in the communication between these groups and those who are key to disseminate any information quickly.

The above would allow any organization to support its decisions for the fulfillment of its objectives, as well as a better organizational behavior and adequate control of trends. In this sense, the algorithm developed allows identifying who influences whom, who is influenced and who are the most influential people, which evidences and validates the case study presented, where the fulfillment of the research hypothesis is demonstrated.

The main limitation of the study is the application scenario, which uses a sample that is not representative of the population, so the results obtained cannot be generalized. Nevertheless, the research can be used as an updated reference in the analysis of the phenomenon of authority and social influence. Likewise, the results can be applied in similar conditions to evaluate, measure and analyze the authority and influence exercised by users in social and professional networks.

Future work in this area of knowledge and application should focus on developing new methods that measure, from different perspectives and more comprehensively, the social influence of users. Some of those that characterize influence are coherence, empathy, reciprocity, scarcity and social approval, which have origins in psychology and sociology. Similarly, the use of the algorithm developed on large data sets is recommended to evaluate its feasibility and applicability for decision making, in relation to the analysis of the authority and influence of users.

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